Notes Regarding Dupuis', 1910, Elements of Astronomy: Principally on the Mechanical Side

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In 1910, N.F. Dupuis wrote The Elements Of Astronomy: Principally On The Mechanical Side Intended For Engineering Students.

The book can be found on Archive.org `https://archive.org/details/cu31924031322203`.

It found the book interesting because it's reads like a popular introduction into astronomy but has some mathematics, to make the book feel more substantive. Even out the outset of the book the author tries to quantify even the simplest things.

> The number of stars visible to the unaided eye is very deceptive. To the superficial observer this number appears to run far into the thousands, but an actual count will show that a normal eye cannot see more than from 1,000 to 1,500 at any one time. And as we can see only one-half of the whole heavens at once, the total number of stars visible in both hemispheres will vary from 2,000 to 3,000, depending on the quality of the observer's eye. (Page 3 and 4)

The book presents facts to make the reader feel more comfortable with astronomy.

> In fact, the great constellations are practically the same to us as they were to the ancient Babylonians 5,000 years ago.(Page, 5)

The book is about practicality and not mystical. From the section on why the earth is not flat. A simple statement that uses real world example and is directly relevant to navigation.

> The highest mountain in the world, Mount Everest, cannot be seen at a greater distance than about 200 miles, and then it is only the top that appears above the distant scene.(Page, 6)

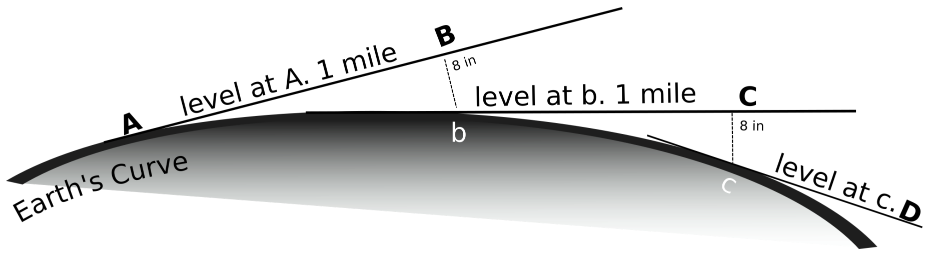
There is also some interest jargon in the book that feels very Victorian.

> When we stand upon the shore of the ocean, or of some great lake like Ontario, we see at some distance a line where the sky and the water appear to meet. This is the *offing*, and it marks the extreme distance at which the surface of the water is visible.(Page, 7)

It's nice to read about a time when canals were the main preoccupation of engineers. Who knew they dropped 8 inches a mile.

> Some interesting problems in engineering arise out of this curvature of the surface of a body of water and these problems in themselves bear evidence to the earth's rotundity Thus a "level line" as determined by the engineer's level is tangent to the earth's surface at the point of observation, and if the earth were a plane this line would coincide with a water surface for any distance. But if such a line be sighted across a lake several miles in width it is quite apparent that

the height of the line above the water is greater upon the further side of the lake than it is at the point of observation.

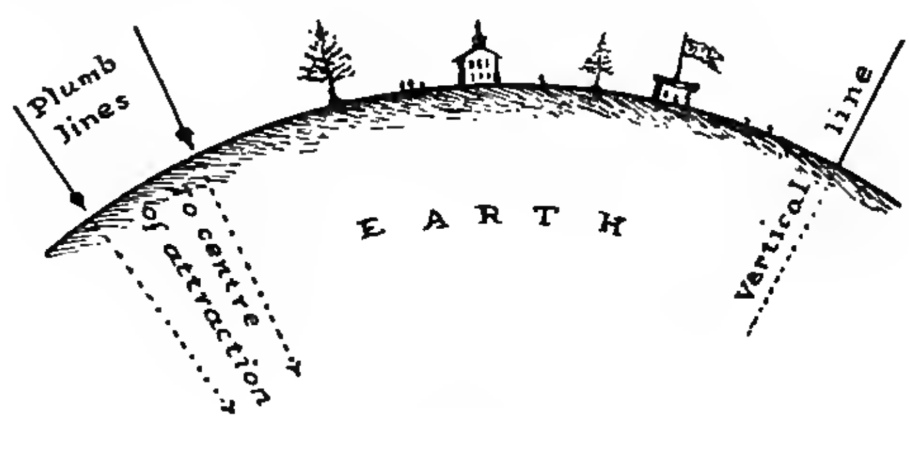


Again, if the bottom of a canal, several miles long, be made parallel to the engineer's "level line," it is well known that the water in the canal will not be of uniform depth throughout, but will get shallower as we recede from the starting point. And it is necessary in order to prevent this, to drop the bottom of the canal eight inches at the end of the first mile, and from this point to adopt a new level line, and to continue this process. Thus if A be the starting point and AB be one mile in length and parallel to the level line at A, the water at B will be 8 inches from B to b and make b a new starting point for the level line bC, etc.

Of course in practice the engineer avoids the abrupt drops of 8 inches at B, C, etc., by distributing them throughout the whole extent of the canal.(Pages 7 and 8)

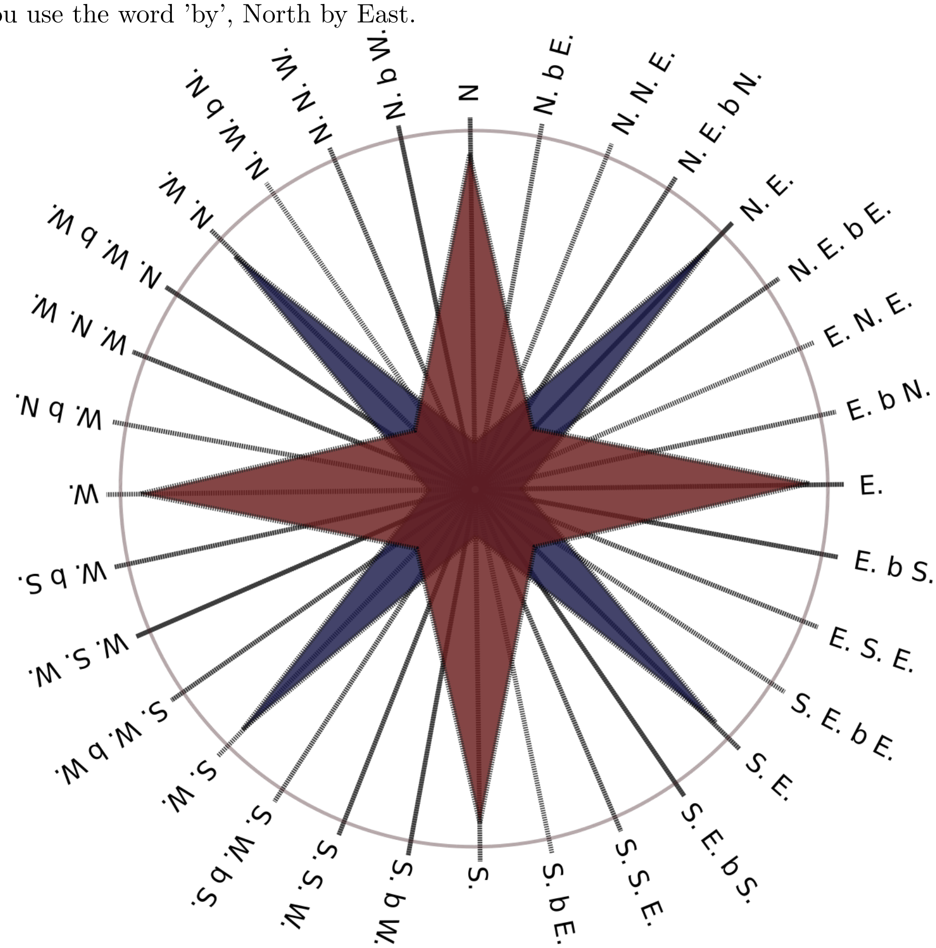
Another interesting fact, every 69.1 miles is 1°.

From the nature of the case, it is readily seen that the plumb-lines at any two places upon the earth's surface cannot be parallel; for even in case of antipodes, or points on the earth exactly opposite, the lines, although parallel, are stretched by their bobs in opposite directions.(Page, 10)



Two plumb-lines 69.1 miles apart make an angle of 1°with one another, and if they are one mile apart the angle between them is about 52".(Page, 10)

The 32 point compass rose is given. I'm not sure why the author included it. It has more to do with sailing than with astronomy. But I do like the look of the compass rose. The author's explanation is a bit lacking. It would be better to say that mariners prefer the compass rose because it is easier to draw. You sub-divide the cardinal points of North and East and you get North East. Have of North and North-East gets you North-North-East. With the third division you use the word 'by', North by East.



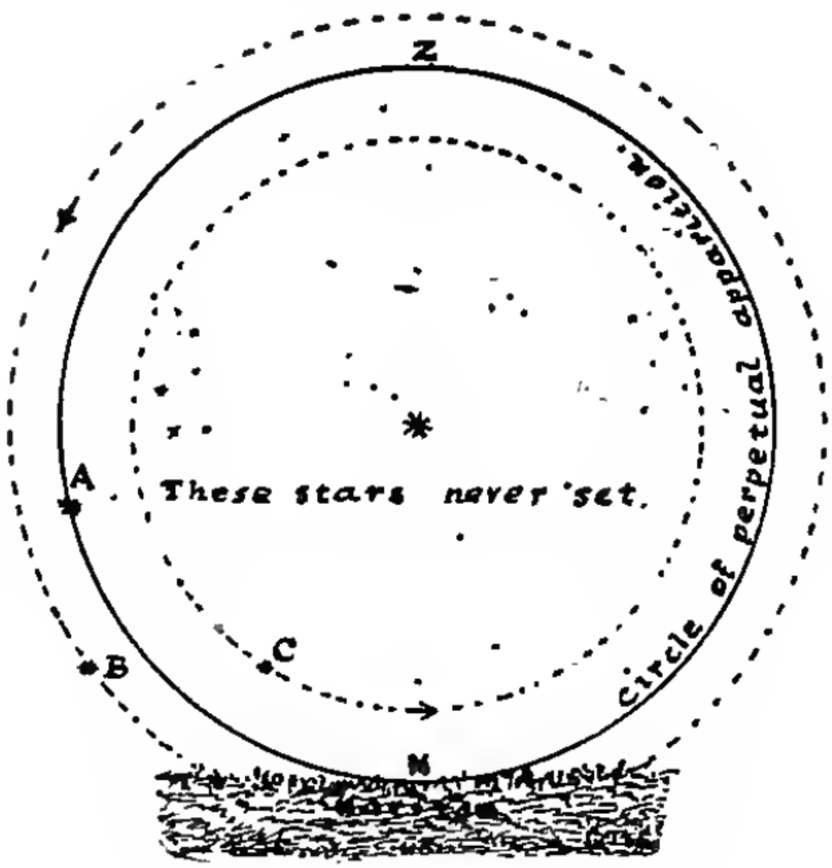
The Horizon is divided into four equal parts by the four cardinal points of the compass, North, East, South and West. Each of these parts is again divided into 8 equal parts, giving in al 32 points of the mariners compass. The names of these points, staring from the north are: North, North by East, North North-East, Northeast by North,

> North-East, Northeast by East, East Northeast, East by North, East, East by South, East Southeast, Southeast by East, Southeast, Southeast by South, South Southeast, South by East, South, South by West, South Southwest, southwest by south, Southwest, Southwest by West, West Southwest, West by South, West, West by North, West Northwest, Northwest by West, Northwest, Northwest by North, North Northwest, North by West.
>
> The Horizon is divided into four equal parts by the four cardinal points of the compass, North, East, South and West. Each of these parts is again divided into 8 equal parts, giving in al 32 points of the mariners compass. The names of these points, staring from the north are:
>
> The mariner then divides each of these 32 points into four equal parts called quarter points, thus making 128 quarter points in the circle of 360°, so that the value of a quarter point is 2°48'45".(Page, 14)

Astronomical language is always great. Here is presented the *Circle of Perpetual Apparition*.

> As has already been pointed out, the relative positions of the stars are the same from year to year, so that if the stars move materially they must move as a whole, and not individually.
>
> Upon an starlit night, preferably when the moon is absent, let one take up a position in which he can command a fairly unobstructed view of the horizon, and let him direct his view, at first, to the northern sky. We will suppose that he is at the latitude 45°north.
>
> He will observe about half-way between the north point of the horizon and the zenith, a fairly bright star which stands pretty much alone, having no equally bright stars within some distance of it, and in line with the two stars, in Ursa Major, known as the pointers. This is *Polaris*, or the *north star*, or the *pole star*.
>
> Our observer, by continuing his observations for several hours, will notice that all the stars of the northern sky appear to move in circles having Polaris near the centre, the direction of motion being opposite to that of the hands of his watch.
>
> Those stars at the proper distance from the pole, in this case 45°, as represented by the circle *A*, will graze the northern horizon at *N* at their lowest point, and will pass through the zenith, *z*, at their highest.
>
> But stars at less than 45°from the pole, as represented by the circle *C*, never reach the horizon and therefore never set, and can be seen, by means of a telescope, at any time when the northern sky

is unclouded. Hence the circle A is called the *Circle of Perpetual Apparition.*



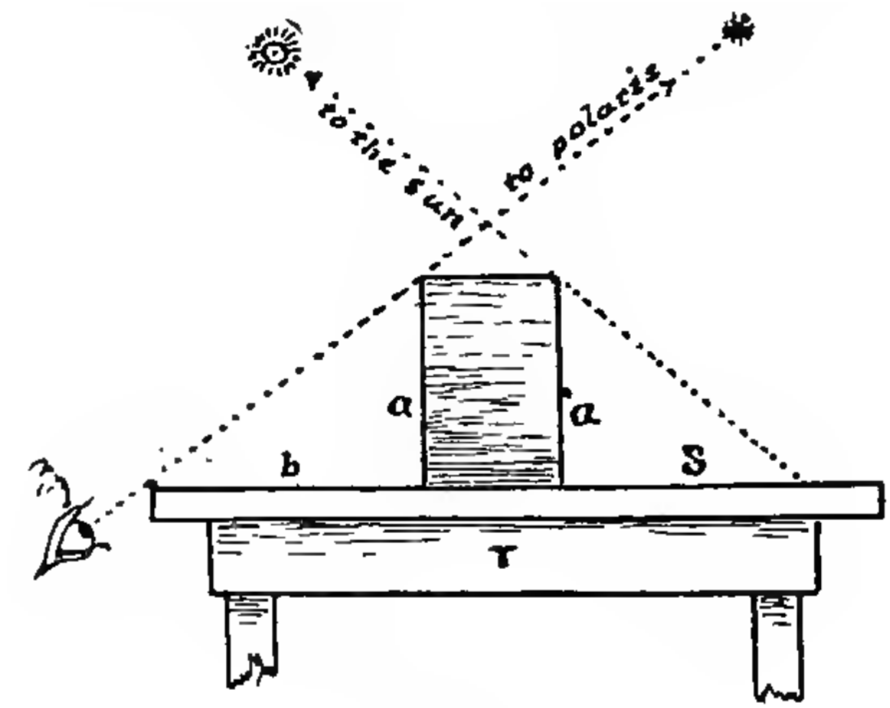
Of the stars which rise and set, those which rise near the east point of the horizon remain about 12 hours above the horizon and set near its west point; those which rise south of east remain less than 12 hours above the horizon and set south of west. And the farther south a star rises, the farther south it sets and the shorter time it remains above the horizon. And these observations are for 45°north latitude, or for an observer situated half-way between the earth's equator and its north pole.

If the observer goes southward the pole star descends towards the north point of the horizon, and the circle of perpetual apparition gets smaller, no longer reaching to the zenith; and if he goes northward the opposite change takes place, that is, the pole star rises and the circle of perpetual apparition grows larger and includes a greater number of stars. Altogether similar phenomena would be observed by a person living south of the equator.(Pages, 18-20)

Here is a cute little experiment outlined in the book.

Thus we can find our latitude upon the earth, by finding the altitude of the pole, or of the sun on the meridian, if we know the sun's declination, and this is given in the Nautical Almanac for every day in the year. The following experimental observation my be carried out by the beginner:

T is a table which is placed with its longer edge as nearly north and south as can be arranged, and is then carefully levelled b is a box, with right-angled corners, so placed upon the table that its horizontal edges are parallel with those of the table. By placing one's eye, E, in the proper position the edges of the tabe and the box may be brought in the line with Polaris, $($P), by moving the box on the table.



The carefully measuring the lengths a and b, we have a/b=the tangent of natural tangents, is the altitude of Polaris and approximately the latitude.

Thus if a=12 in. and b=15 in., tangent of altitude of Polaris=0.8, the altitude itself is 38°40'; and this is approximately the latitude.(Pages, 27-28)

The author explain how to convert degrees into local time, on pages 29 and 30.

$$Divide \quad the \quad \left|\begin{array}{c} \circ \\ \prime \\ " \end{array}\right| by \quad 15; \quad call \quad the \quad quotients \quad \left|\begin{array}{c} hr. \\ min. \\ sec. \end{array}\right|$$

$$Multiply \quad the \quad remainder \quad from \quad the \quad division \quad of \left|\begin{array}{c} \circ \\ \prime \end{array}\right| by \quad 4,$$

$$and \quad call \quad the \quad products \left|\begin{array}{c} \circ \\ \prime \end{array}\right| Add \quad results.$$

Example. To change 76°28' 52" to time:

Degrees	76°	28′	52"
Divide by 15	5*h*.	1*m*.	3*s*.
Reminder	1	13	7
Multiplie by 4		4*m*.	52.47*s*
Sum together	5*h*	5*m*	55.47*s*

[*These are the coordinates for Kingston, Ontario, Canada and the time difference between the local time (Kingston) and Greenwich Mean Time (GMT).*]

To change time into angle, multiply h.m.s. each by 15 and call the products °' ", and reduce.

Example. To express 5h 5m 55.47s in angle:

Angle	5*h*	5*m*	55.47*s*
Multiplying by 15	75°	75′	832.05"
Reduce	76°	28′	52.05"

The advantage of expressing the longitude of a place in time instead of in angle is apparent from the fact that the longitude expressed in time tells us at once how much the local time of the place is behind that of the first meridian.

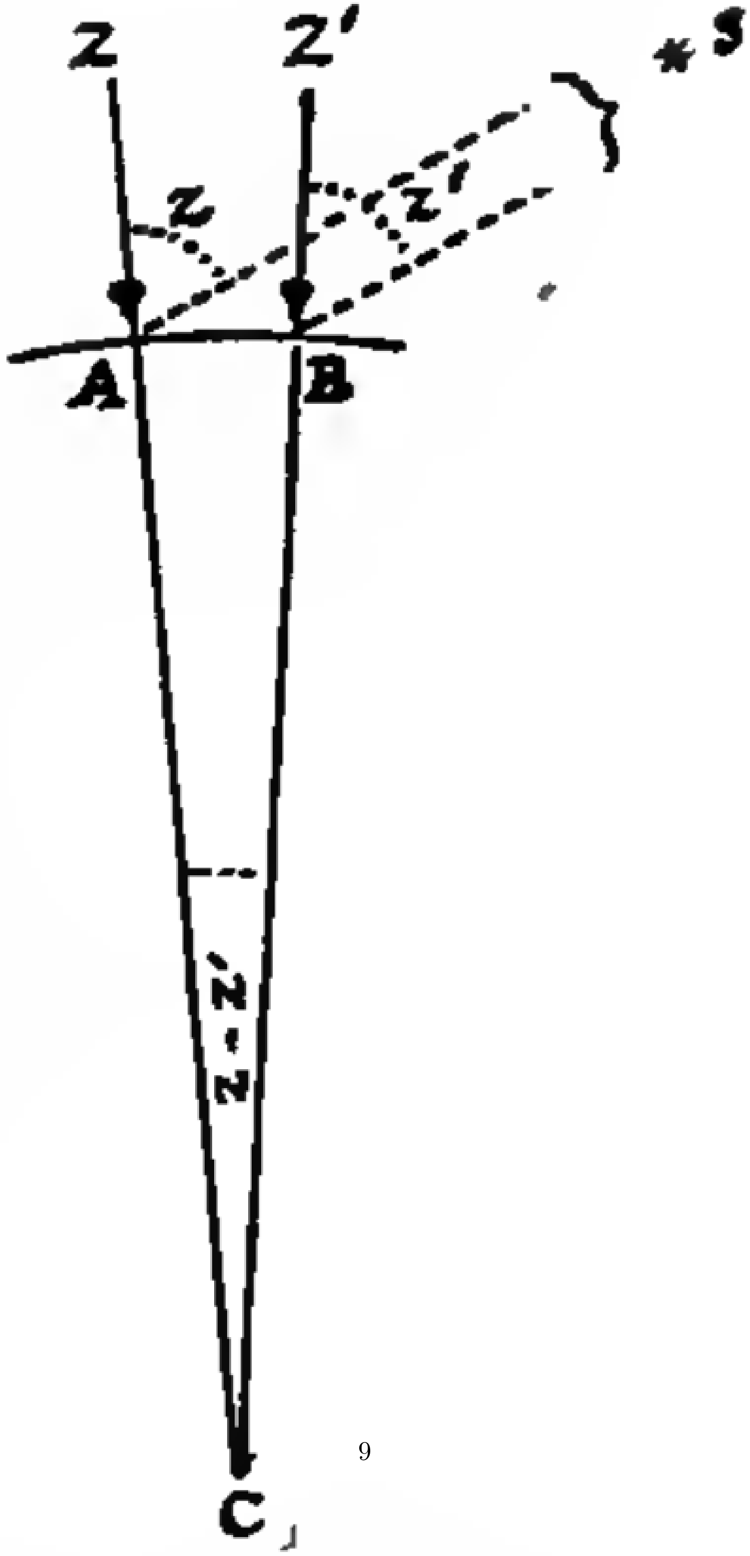
Thus if the longitude of a place be 8h 20m then the time at the place is 8h 20m slower than at the first meridian; so that if the time at the first meridian is 10h 50m a.m., the time at the given place is 2h 30m a.m.

Taking the meridian of Greenwich as the first meridian, the longitude of Kingston is 5h 5m 55.5s, which is equivalent to saying that the local time at Kingston is 5h 5m 55.5s behind that local time at Greenwich.

The Formula $s = r\theta$ (Pages, 33-4)

In this well-known trigonometrical formula, which we shall have occasion to use quite frequently, s denotes the length of an arc of a circle, in any convenient length-unit; θ is the radian measure that this arc subtends at the centre; and r

is the radius of the circle; and in this formula these are so connected that if any two are given the third can be found.


Z
Z'
*S
z
z'
A
B
z-z'
C

Let Z and Z' be the zeniths of two places A and B situated on the same meridian. Then if ZA and $Z'B$ be produced downwards they will meet at the centre of the circle of which the meridian AB is a part. Let S be a star at its culmination. The angle ZAS is the zenith-distance of the star as seen from A, and $Z'BS$ is the zenith-distance as seen from B, and these two angles are got by observation on S when on the meridian.

Then it is easily seen that the difference of these zenith-distances is the angle at C, or θ.

The next thing is to measure the arc AB. this is usually done on some extensive tract of level country, and every known refinement in the process of length measurement is employed. This measure gives the value of s and thence r is found. For a mean value it is necessary to choose the places A and B at mean latitudes, as the extremes of radii are at the equator and the pole.

The Earth's Radius (Pages, 34-5)

Taking now a practical case: In England at latitude about 52°N. it was found that a distance of 364,971 feet subtended an angle of 1°, or $\pi/180$radians. Then r=364971x180/π=20911300 feet = 3960.4 miles. And 3960 miles may be taken as a close approximation to the earth's mean radius. This value for the radius of a spherical earth gives the following results:

Length of the equator, 24884 miles.

Length of 1°in latitude, 69.1 miles.

Length of 1' in latitude, 1.517 miles, or 6081 feet.

Length of 1" in latitude, 101 feet very nearly.

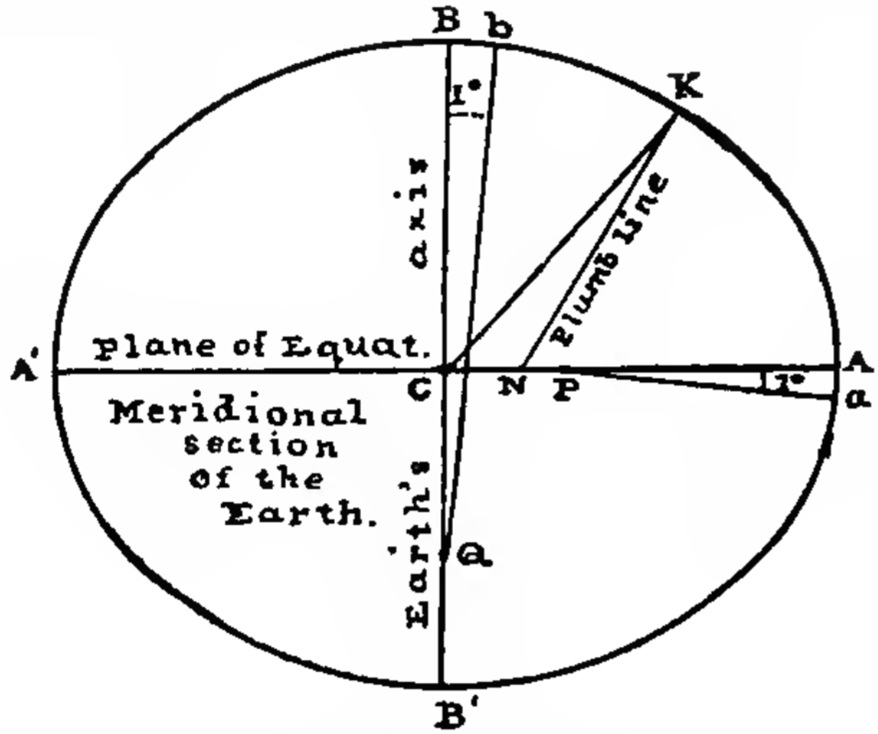
This gives for the velocity of a point on the equator, owing to the axial rotation of the earth, about 1040 miles an hour, or over 17 miles a minute. But how do we know that earth is an oblate spheroid and not a sphere, for as yet we have had only theory?

A Spheroidal Earth (Pages, 35-7)

Measurements of arcs, by measuring rods and triangulation, have been carried out in many parts of the world and in different latitudes, and in the following table we have some of the results:

Country.	Latitude.	Feet in 1°of lat.
Peru	1° 31'S.	362808
India	16° 18'N.	363004
America	39° 12'N.	363786
France	44° 51'N.	364535
England	52° 35'N	364971
Sweden	66° 20'N	365782

This table shows clearly that for a given constant, 1°, the length of the arc increases as we go from the equator towards the pole. And considering the arcs as small parts of circles, we see that near the pole the meridian belongs to a greater circle than when near the equator, or, in other words, the meridian is less curved at the pole than at the equator.



Let *AB A'B;* be a section through a meridian of the earth considered as an oblate spheroid. Take *B, B'* the points of the shorter axis as the poles, and then *A* and *A'*, the endpoints of the longer axis, will be points on the equator.

Then it is evident that the meridian is less curved at B than at A, as it should be. And if bQ, aP be normals to the meridian at b and a, and be so taken that the angles BQb and APa are equal and each 1°say, then it is evident that the arc Bb at the pole is longer than arc Aa at the equator: and as a consequence BQ is greater than AP.

If we know Aa and Bb, we can find AP and BQ by the formula $s = r\theta$. And what we wish finally to find are the radii AC and BC, or a and b.

From the table, by interpolation, we find Aa to be 362740 feet, and Bb to be 366410 feet.

Thence we obtain the values of Aa and BQ; and finaly by means of a property of the ellipse, which cannot well be given in this work, we get
AC=a=3962.8 miles
BC=b=3949.5 miles;
and these are respectively the values of the equatorial radius, and of the polar

radius of the earth.

This gives a difference of 13.3 miles between the equatorial radius and the polar radius of the earth, or 26.6 miles between the two corresponding diameters.

Not this 26.6 miles is only about one-three-hundredth part of the whole diameter, and this is expressed by saying that the earth's oblateness is one-three-hundredth, of 1/300.

To form a proper conception of this we may represent a meridian of the earth by drawing an ellipse, or oblong circle, having its diameters respectively 6 in. and 6.02 in.; and if such a figure were accurately drawn it would require careful measurement to prove that it was not a circle.

This protuberance of the equatorial parts is a positive proof that the earth rotates on its axis.

This oblateness, small as it is, has some interesting results. (1) *Two latitudes.* The more important result to be mentioned at present is that every place on earth, unless it be at a pole or on the equator, has two latitudes. We have defined the latitude of a place as its angular distance from the plane of the equator. Now [in the above figure], let K be a place on the meridian. The plumb-line ZK produced downwards does not pass through C, the centre of the earth, but through a point N on the plane of the equator. The definition gives either of the angles KNA or KCA as the latitude of K. But as the altazimuth is adjusted by the level, which is equivalent to the plumb-line, the angle KNA is the one determined by the instrument. This is accordingly called the observed or *apparent latitude*, while the angle KCA is the *geocentric latitude*, or the latitude as seen from the centre of the earth. The difference, the angle CKN, is called the *correction* of the latitude, or the *angle of vertical.* It will be sufficient here to say that the greatest value of this angle is about 11'30", which occurs at the latitudes about 45°. Apparent latitude is always greater than geocentric, except as before mentioned.

(2) As the pole is nearer the centre of the earth than the equator is, attraction is stronger at the pole than at the equator. So that if a body which, by spring balance, weighs 194 pounds at the equator, be taken northward, it continually increases in weight until the pole is reached, when it weighs 195 pounds.

Also, the time taken by a pendulum of given length to make one oscillation depends upon the pull of gravity upon the bob of the pendulum. And on account of the earth's oblateness a pendulum clock that keeps correct time at the equator would gain about 4 1/2 minutes daily if taken to the pole. As the surface of the sea may be considered as representing the form of the earth, this form might be determined by timing the oscillations of a standard pendulum made at the seashore in different latitudes.

It is interesting to follow the sequence of small effects which result from the revolution of the earth upon its axis. Some of these have been given in what precedes; others will follow in the proper place.

As the oblateness of the earth is so very small it is quite sufficient for general purposes, as has been said before, to regard the earth as a sphere with a radius of 3960 miles; but where accuracy is required the oblateness must be taken into account.

Parallax (Pages, 38-9)

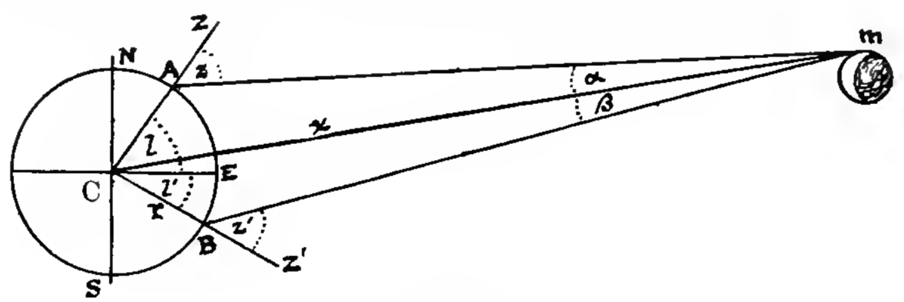
...and we may accordingly define parallax as *an apparent displacement of an object due to a real displacement of the observer.*

Some of the stars have parallaxes of less than 1" when the diameter of the earth's orbit, a length of over 180 million miles, is taken as a base. And even with this enormous base the majority of the stars have no appreciable parallax.

Limb. When a heavenly body presents a circular disc the edge of the disc is called the emphlimb, and different parts of the edge are distinguished by naming them, as the upper limb, the eastern limb, etc. As it is not practicable to observe the centre of the disc, there being no point or mark to distinguish it, observations are made upon the limb intsead. Thus, to find the altitude of the sun, we measure the altitude of its upper limb, and apply a correction for its semi-diameter.

Moon's Distance (Pages, 42-3)

If we could find the moon's horizontal parallax directly by observation, nothing would be easier than to find the moon's distance by application of the formula $s = r\theta$. This might be done under certain special arrangements, but as observatories are situated, not at special points, it is more practicable to follow another course, as follows:



C is the earth's centre and m is the moon's northern limb, and the problem is to find Cm. Let A and B be two observers on the same meridian, or nearly so, as at Berlin and the Cape of Good Hope. And let their latitudes be denoted by l and l'. Then the angle ACB is $l - l'$. The observers measure the zenith distances ZAm and $Z'Bm$ respectively. Denote these by z and z'. Also let r be the radius of the earth, and denote the distance Cm by x, the angle AmC by α, and BmC by β.

$\sin\alpha = (r/x)\sin z$, and $\sin\beta = (r/x)\sin z'$.

$\therefore \sin\alpha + \sin\beta = (r/x)(\sin z = \sin z')$.

But by a well-known formula,

$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$

And $\alpha - \beta$ is necessarily a small angle, and $\frac{1}{2}(\alpha - \beta)$ will not exceed $10'$, so that $\cos\frac{1}{2}(\alpha - \beta) = 1$ without any appreciable error. Thence by substitution,

$x = \frac{1}{2}r(\sin z + \sin z')/\sin\frac{1}{2}(\alpha + \beta)$

But $ACBM$ is a quadrangle and the sum of its angles is four right angles; and these angles are

$\alpha + \beta$, $180° - z$, $180° - z'$, and $l = l'$.

$\therefore \alpha - \beta = z + z' - l - l'$.

And finally,

$x = \frac{1}{2}r(\sin z + \sin z')/sin\frac{1}{2}(z + z' - l - l')$.

Illustration. Suppose for illustration that $l = 30°$ N., $l' = 20°$S., and that z and z' are found to be $35°$ and $15°48'$ respectively. Then $\sin z + \sin z' = 0.84586$,

$\sin\frac{1}{2}(z + z' - l - l') = \sin 24 = 0.00698$.

And $x = \frac{1}{2}r \times 0.84586/0.00698 = 60.6r$.

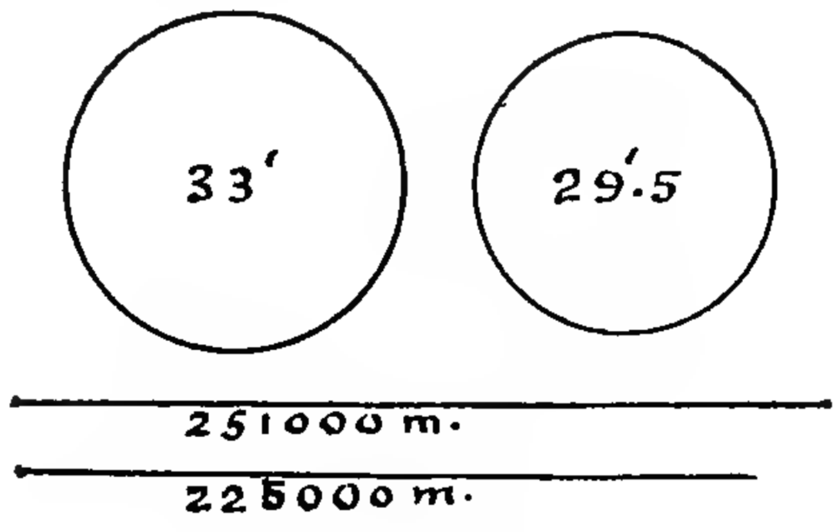
This makes the moon's distance from the centre of the earth to be 60.6 times the radius of the earth, or about 240,000 miles if the earth's radius is taken as 3960 miles. And this is about correct.

Moon's Distance (Pages, 44-45)

The angle subtended by the disc of the moon, or the moon's angular diameter, is bout $31'$, and its distance is 240,000 miles. Hence the moon's diameter $=$ $(31/60) \times 240000 \times \pi/180 = 2160)$miles. [31/60 because 31 minutes needs to be converted to degrees. Multiplied by $\pi/180$ to convert to radians.]

The moon's diameter is thus three-elevenths of that of the earth. And as the volumes of spheres are proportional to the cubes of their diameters we find the volume of the earth to be about 50 times that of the moon.

Variations in moon's distance The average angular diameter of the moon is about $31'$, but this angle is variable ranging from $29\frac{1}{2}'$ to $33'$, as represented in the diagram.

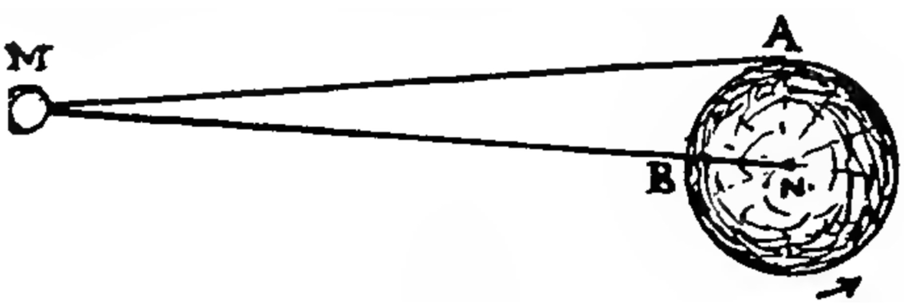


Now, as the moon does not change its linear diameter, it must change its distance from the earth; and by the application of the formula $s = r\theta$ we readily find that when the angular diameter is $29\frac{1}{2}'$ to $33'$ the distance is 251,000 miles and when $33'$ the distance is 255,000 miles. These distances are represented by the relative lengths of the lines in the diagram.

The mean of these two extremes is 238,000 miles, and this we shall take as the *average* distance of the moon.

The rising and setting of the moon, as of the sun and stars, is due to the rotation of the earth on its axis.

Moon's Diurnal Augmentation. (Pages, 45-47)



Let N be the north pole of the earth, and M be the moon, and for simplicity of explanation suppose the moon to be on the celestial equator. To the observer at A the moon is on the horizon and is rising. When, by the earth's rotation, the observer comes to B he is nearly 4000 miles nearer to the moon than he was at A, and the moon must appear larger than at A. This increase in the moon's apparent diameter is known as the *moon's diurnal augmentation*, as it is repeated in every lunar day. Its greatest amount is about one-half a minute of angle.

Mass-centre. (Pages, 49-50)

Now various considerations have shown that the earth is about 80 times as heavy as the moon, being apparently formed of denser more compacted material, so the mass centre of the two bodies is $\frac{1}{80}$ of 240,000 miles, or 3000 miles from the earth's centre, or at a point 1000 miles below the its surface. [Earth's radius is 3960 miles] Generally speaking, then, it would savor of pedantry to say that the moon does not revolve about the earth, but about a point 1000 miles below its surface, although the later statement would be the correct one. And, in fact, it would be quite right to say that the earth revolves about the moon, if anything in the way of explanation were to be gained thereby, for in either case outside phenomena would be but little, if at all, interfered with.

We see, then, that the path of the earth's centre as it moves along in its orbit about the sun, is not a smooth line or a curve, but a wavy one, in which a wave is completed at every revolution of the moon. This throws the centre out to the extent of nearly 3000 miles, first to one side of the ideal orbit and then to the other, besides alternately retarding and accelerating its motion.

The Sun's Distance. (Pages, 54-55)

As we are not yet prepared to show how the sun's horizontal parallax is found, we shall in the meantime assume its value to be 8".8 leaving it to a future occasion to show how so small an angle is determined.

As the sun's horizontal parallax is the angle subtended by the earths radius as seen from the sun, we apply the formula $s = r\theta$ and obtain:

$3960 = D \times 8.8 \times \pi/(3600 \times 180)$

whence D = 92,900,000 miles, nearly.

And for the average distance of the sun we shall take D=93,000,000 miles.

This is easily show to be nearly 400 times the distance of the moon.

For, if the earth were represented by a small circle one-tenth of an inch in diameter, the moon would be represented by a circle only one thirty-sixth of an inch in diameter, at the distance of three inches from the earth the the sun would be a circle of 10.8 inches in diameter at a distance from the earth of about 98 feet.

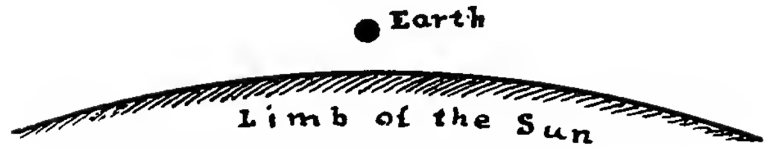
Thus, as the sun is about 400 times as far from the earth as the moon is, if the distance of the moon be represented by one-tenth of an inch the sun's distance would become 40 inches, and that of the most distant planet, Neptune, about 100 feet, measures altogether impracticable in graphic illustration.

The Sun's Diameter. (Pages, 54-55)

Knowing the mean distance of the sun, it is an easy matter to find the sun's angular diameter and then to find its diameter in miles by using the formula $s = r\theta$. For r is 93,000,000 miles, and θ is the radian measure of $32'$, which is the mean angular diameter of the sun, and this gives about 850,000 miles for the sun's linear diameter.

This is nearly 108 times the diameter of the earth, so that it would require 108 earths placed side by side to reach across the sun.

As a consequence, to represent truthfully the comparative sizes of the earth and the sun, we may take a small circle one-twelfth of an inch in diameter to represent the earth, and a circle of 10.8 inches in diameter for the sun, as in the diagram where only a part of the sun's disc is shown.



The Sun's Volume. (Pages, 56)

As the volumes of the spheres are proportional to the cubes of their diameters, the volume of the sun is $108^3 = 1,250,000$ times, nearly, that of the earth; in other words, it would require one and a quarter millions of bodies like this earth to make one body equal in bulk to the sun.

Sun's angular diameter. (Page, 57)

The angular diameter of the sun, like that of the moon, is not constant, but varies from $32'31".8$ at its least, giving a difference of $1'4".6$.

This shows that the distance of the sun is variable. Working out the greatest and the least distance as given by the least and greatest diameters, we get:

Date.	Angular diam.	$\odot$'s distance .
Jan. 1st	$32'36".4$	91,197,000 miles
July 1st	$31'31".8$	94,312,000 miles

Earth's Orbit. (Page, 57)

The earth's orbit is an ellipse, with the sun at one of the foci, thus satisfying the first law of Kepler. The distance of the sun from the centre of the ellipse is about 1,600,000 miles.

The point where the earth is nearest the sun is the *perihelion*, and the point at which it is the most distant is the *aphelion*.

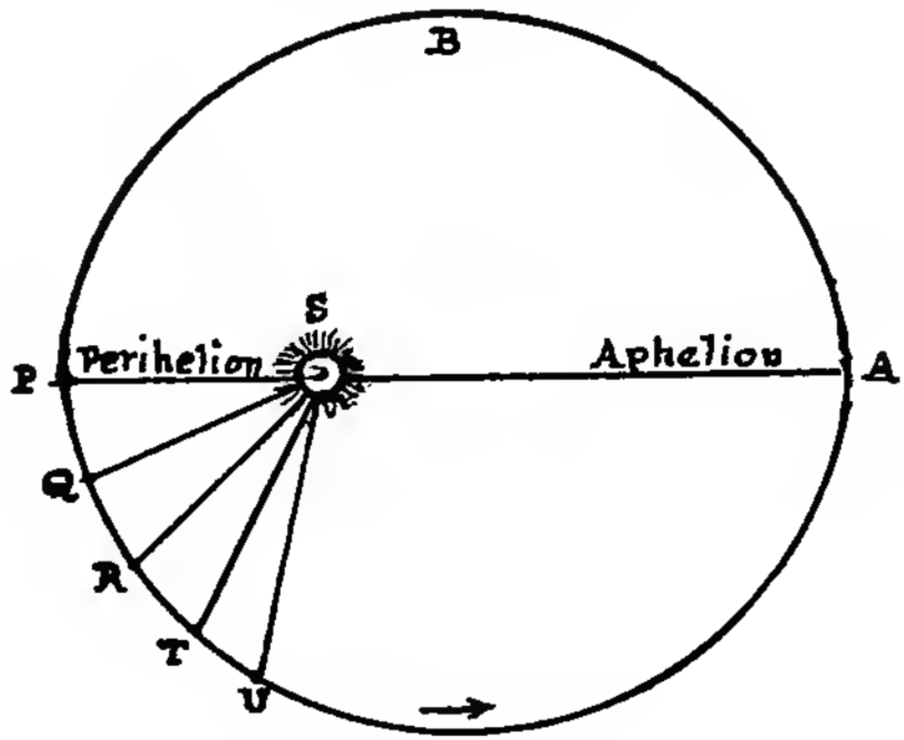
At present the earth is in perihelion on Jan. 1st, and in aphelion on July 1st. And thus, strange as it may appear, inhabitants of the northern hemisphere are nearer the sun in the depths of winter than they are in the warmth of summer. For those in the southern hemisphere the case is, of course, just the reverse.

This may have some effect on the seasons in different hemispheres in the way of making the extremes of temperature in the southern hemisphere somewhat greater than in the northern one, but such effect, if any, does not appear to be very strongly marked.

Kepler's law, II. (Pages, 57-58)

In orbital motion of one body about another, as the moon about the earth, or the earth about the sun, etc., the line joining the two bodies is called the *radius vector*; and it is a physical principle in astronomy, known as Kepler's second law, that the radius vector sweeps over equal areas in equal times.

Let ABP represent the earth's elliptic orbit, necessarily much exaggerated in eccentricity in order to strengthen the illustration, and let S be the sun, P the perihelion and A the aphelion.



Starting from P, let the earth arrive at Q at the end of one week, say, at R

at the end of two weeks, at T at the end of three weeks, etc. Then Kepler's law II tells us that the sectorial areas PSQ, QSR, RST, etc., are all equal.

But SP is less than SQ, and SQ is less than SR, which is again less than ST, and each is less than SA.

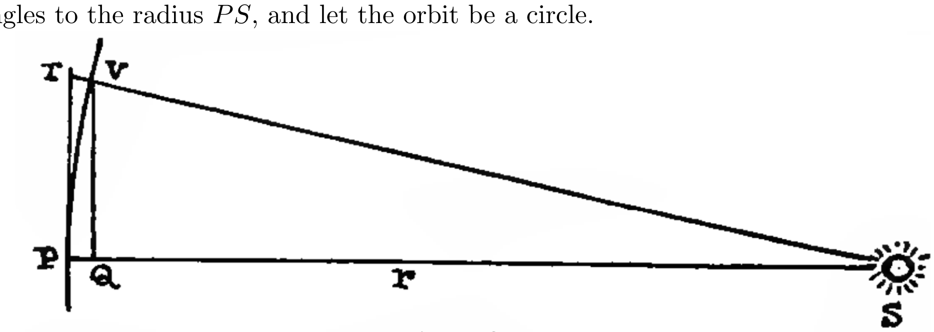
Hence PQ is greater than QR, QR is greater than RT, etc.

But as these arcs are each described in the same time of one week, the earth must move more rapidly from P to Q than from Q to R, etc. Or, in other words, the earth moves most rapidly at the perihelion P, and its motion is gradually retarded until it reaches aphelion at A, at which point it moves most slowly. After passing A its velocity is gradually accelerated until it arrives at P again

Consistently with this theoretical inference, observations made upon the sun's apparent daily motion along its path in the heavens - that is, the earth's real motion in its orbit - gives $61'9"$ on January 1st, and $57'11"$ on July 1st, the mean daily motion for the whole year being $59'8".2"$.

Relation between Period and Distance of a Planet. Kepler's law, III. (Page, 146)

Let S be the sun and let the planet at P be moving in the direction PT at right angles to the radius PS, and let the orbit be a circle.



The attraction of the sun on the planet pulls it in the direction PS, and as the orbit is a circle, PQ may be taken as the measure of the sun's attraction and be denoted by f, while QV, denoted by v, will represent the planet's velocity in its orbit; for however near V is to P the motions PQ and QV brings P to V.

But by an elementary theorem on the circle $PQ(2r - PQ) = QV^2$ And PQ being very small in comparison with PS and QV, its square may be rejected when V is near P, so that at the limit $PQ = QV^2/2r$, or $f = cv^2/r$ where c is a constant.

But if t be the time of revolution about the sun, $v = 2\pi/t$. And m denoting the mass of the sun, $f = c_1m/r^2$ where c_1 and m are constants.

Where eliminating f and v, $r^3/t^2 = a$ constant.

That is, for a planet moving in a circle *the cube of the radius of the orbit is proportional to the square of the time of revolution.* As as all the planets move in orbits which are nearly circles, and their masses are very small as compared to that of the sun, we may make the general statement that with any two planets *the squares of their periodic times are proportional to the cubes of their mean distance from the sun*; and this is Kepler's law III.