Notes Regarding Cullimore's, 1915, The Use of the Slide Rule; Accuracy and Significant Figures

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I came across Cullimore's, 1915, book The Use Of The Slide Rule and found the introductory chapter 'Accuracy and Significant Figures' most interesting. The book can be found on Archive.org `https://archive.org/details/useofsliderule00cullrich`. I have typeset the passages using LaTeX, for better clarity, latter on in the PDF. First I'll demonstrate the authors own example using the R programming language. Opening an R terminal.
> Length = 101.13
> Breath = 7.34
> Depth = 9.3
> Box = Length*Breath*Depth
> Box
[1] 6903.336
We see here that the volume is 6903.336, which is not correct, too much precision gave us he wrong answer. That is because Depth is only accurate to one decimal place.

The following is chapter I, from Cullimore.

I. Accuracy and Significant Figures

It is absolutely necessary for a proper and efficient use of the slide rule that the operator should have a clear and definite idea of the conditions under which the rule may be used to advantage. Even among engineers, the idea is often expressed that the slide rule is inaccurate, because, in the hands of a reasonably expert operator, the rule will give results accurate only to 1/10 of 1%. It should be borne in mind that the rule is inaccurate in exactly the same way that a four place table of logarithms is inaccurate.

In a very large class of engineering calculations, however, results which are well within the allowable error may be computed by the slide rule. The question most often arising is whether the rule is adapted to a given problem or not. To answer this, requires a knowledge of the proper use of significant figures and a close inspection of the data of the problem, together with a knowledge of the means employed in obtaining these data.

By a significant figure we mean any figure which is significant in that it gives some real information as regards the quantity which is represented Thus:

18700.1	has	six	significant figures.
13.7302	"	six	"
0.0032	"	two	"
13000	"	two or five	significant figures.

Notice that in the last case an ambiquity arises and that any one or all of the zeros may or may not be significant.

Take the problem of finding the area of a circle whose radius is 4.67 feet, measured with a steel tape. It is easily seen that the recorder of the data meant that he was unable to state the distance closer than 1/100 of a foot. In short, that he felt sure that the distance was nearer .67 of a foot, than either .66 or .68.

He ,therefor, recorded the results 4.67 feet, using three significant figures with an accuracy of one part in 467, or not quite 1/5 of one percent. It should be borne in mind that the number of significant figures expresses accuracy, while the number of decimal places may or may no do so. Thus 761 millimetres is identical with .761 meters,and the decimal place shows nothing. The engineer recording or computing data should force himself to express, by the number of significant figures in the result, the accuracy of that result. The frequent habit of carrying results to a greater number of significant figures than the data warrant comes perilously near to lying with figures; it certainly creates a wrong impression as to the accuracy of the result. Certain mathematical constants can, however, be computed to any number of significant figures; for instance π may be expressed as 3.142 or 3.141592654. On the other hand, certain physical constants are very uncertain, even in the third place. Take, for example, the weight of a cubic foot of water generally given as 62.5 lbs,; conditions of temperature and solution may easily alter the last figure of the three. With recorded data, however, the number of significant figures, as well as their character, gives very definite information. If we say tha light travels 186000 miles per second, we do not mean that it covers 186000 miles to within the smallest fraction of an inch in one second of time, but that the distance covered is nearer 186000 miles than it is 185000 or 187000, and the accuracy 1/186 is expressed by three significant figures. In this particular instance, it will be noticed that the three zeros to the right of the six, may or may not be significant figures. To prevent this ambiguity it has been suggested that the results like above be written $186 \times (10)^3$ which obviates the difficulty.

If, therefore, the slide rule will consistently give results to within 1/10 of one per cent. or to one part in a thousand, we have a right to use it where three significant figures are warranted in the result. The following rule given by Holman should be rigidly observed in all cases: "If numbers are to be multiplied or divided, a given percentage error in one of them will produce the same percentage error in the result." This amounts to saying that all problems, involving data correct to three significant figures only, can be computed advantageously by means of the Slide Rule. The answer is not only near enough, but is accurate as the data warrant. Of course, in cases when the slide rule is used as a more or less rough check on logarithmic or other calculations, these questions of accuracy do no apply. The consideration of a very simple example will serve to illustrate the rule as stated above.

Suppose we wish to compute the cubical contents of a prism of earth. Consider that the horizontal distances have been measured with a tape to the nearest 1/100 of a foot, and that the heights have been measured by a level to the nearest 1/10 of a foot, and the following dimensions recorded:

Length 101.13 ft. Breath 7.34 ft. Depth 9.3 ft.

Multiplying:

101.13
7.34
742.2942
9.3
6903.33606

Now, if the answer be as indicated, we know the contents to within 1/10000 of a cubic foot, or better, with an accuracy of 1/7000000 of one per cent. which is, or course, ridiculous. Suppose the depth, somewhere between 9.25 and 9.34: The correct height might be 9.25 or 9.34 but if tenths alone were expressed, it would be recorded in both cases as 9.3. We see, therefore, that the correct result lies between the following:

742.2942
9.25
6866.221350

742.2942
9.34
6933.027828

We see, then, that actually we know, only, that the result surely lies between 6866.221350 and 6933.027828, but we certainly know nothing more definite than this. If we express the answer first found as 6903.33606, we know nothing about the last seven figures. We are sure of only the first two and the result should have been written 6900. In the light of this we now perform the same multiplication as follows:

$$\begin{array}{r} 101.13 \\ \underline{7.34} \\ 742.0 \\ \underline{9.3} \\ 6900. \end{array}$$

giving the answer 6900, which is as near the true value as we can know by the data recorded.

It will be readily seen that a knowledge of the proper number of significant figures saves an immense amount of time in calculation. This is true no matter what means are used in calculating, whether it be multiplication, logarithms or the slide rule. The operator should accustom himself first to examine the data of a problem and mentally calculate the desired accuracy of the result, as well as the approximate numerical value of that result.