Compute $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x)\frac{1}{x+1}$

Solution,

$$f(x) = \frac{1}{x+1}$$

$$f(x+h) = \frac{1}{x+h+1}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

Multiply the denominator and numerator by a common term to simplify the complex fraction.

$$= \frac{\frac{1}{(x+h+1)}\frac{(x+h+1)(x+1)}{1} - \frac{1}{(x+1)}\frac{(x+h+1)(x+1)}{1}}{h(x+h+1)(x+1)}$$

$$= \frac{\frac{1}{\cancel{(x+h+1)}}\frac{\cancel{(x+h+1)}(x+1)}{1} - \frac{1}{\cancel{x+1)}}\frac{(x+h+1)\cancel{(x+1)}}{1}}{h(x+h+1)(x+1)}$$

$$= \frac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \frac{x+1-x-h-1}{h(x+h+1)(x+1)}$$

$$= \frac{\cancel{x}\cancel{+1}\cancel{-x}-h\cancel{-1}}{h(x+h+1)(x+1)}$$

$$= \frac{-h}{h(x+h+1)(x+1)}$$

$$= \frac{\cancel{-h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \frac{-1}{1(x+h+1)(x+1)}$$

$$= \frac{-1}{(x+h+1)(x+1)}$$

Hence,

$$\lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h \to 0} \frac{-1}{(x+h+1)(x+1)}$$

Replace h with 0.

$$= \frac{-1}{(x+0+1)(x+1)}$$

$$= \frac{-1}{(x+1)(x+1)}$$

$$= \frac{-1}{(x+1)^2}$$